Maximum Stiffness Optimal Control for Linear Actuator Robot Locomotion

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***Abstract*— Linear Actuator Robots (LARs) are frameworks of high-extension linear actuators that can change shape through the coordinated actuation of their members. The lack of robustness of LARs constructed from rigid electromechanical actuators motivates the development of LARs constructed from compliant actuators. However, the limited load carrying ability of compliant LARs requires intelligent control architecture to maximize performance. We propose methods to compute optimal control policies that provide for LAR locomotion while maximizing structural stiffness to external forces.**

I. Introduction

Linear actuator robotics is a class of robotic systems that achieves shape change through the coordinated deformation of a fixed topology of modules [1]–[3]. This shape change can be used for a multitude of tasks including locomotion, manipulation, and shape morphing (Fig. 1). These systems have been built and proposed for space exploration and search and rescue operations to maintain mobility in irregular terrain and access small spaces for exploration and storage [4]–[6]. More advanced LARs could utilize the shape change by forming dynamic and self-erecting architecture, camouflaging by mimicking local topography, and interaction with humans by forming useful objects like ramps, stairs, tables, or chairs.

In previous work, LARs were constructed with rigid electromechanical actuators. This rigid construction has the critical disadvantage of robustness to shock. With no mechanism to absorb and dissipate energy, the rigid electromechanical actuators break or jam after exposure to high impact forces. This is an important limitation for space exploration applications where the system must survive landing on the planet and rolling or falling down in unstructured environments. For this reason, research on LARs has diminished as focus shifted to tensegrity robotics. Tensegrity robots are systems comprised of a set of rigid rods in pure compression connected together by a network of cables in pure tension [7]. These systems have exhibited an inherent compliance that allows them to withstand high impacts [8]. This compliance is an important advancement over the current state of the art LAR. However, tensegrity robots lack the dramatic shape changing ability that LARs have and tensegrity robots are difficult to control due to their complexity and nonlinear coupling between components. Therefore, there is potential for a LAR that is capable of withstanding high impact forces.

LARs constructed from compliant pneumatic reel actuators may offer the robustness of the tensegrity robot with the shape changing ability of LARs [9]. Prototypes of LARs constructed with pneumatic reel actuators do exhibit energy

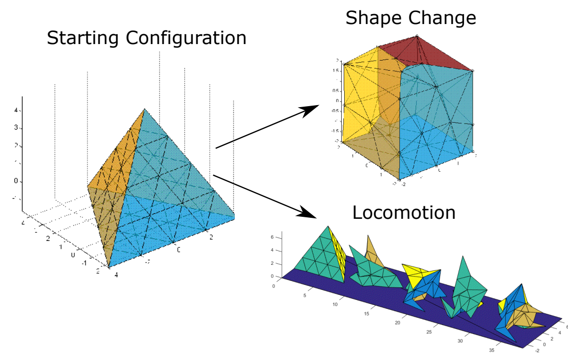

Fig. 1: In this figure, a LAR composed of 108 linear actuators and 34 vertices morphs from a pyramid to a cube (top right), and locomotes with a crawling gate (bottom right).

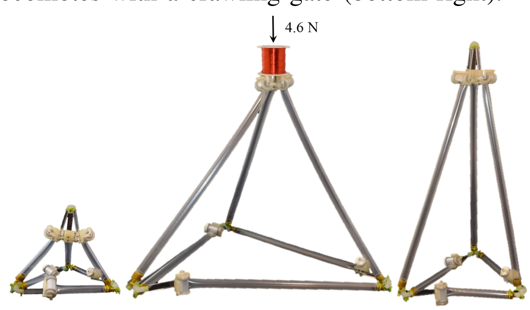

Fig. 2: A tetrahedron constructed from Pneumatic Reel Actuators. Links of this tetrahedron have an extension ratio of 5.4:1. The volume of the robot in its largest configuration is 160 times the volume of the robot in its smallest configuration.

absorbing qualities due to their compliance. However, these compliant LARs do not have as high of a load carrying ability as their electromechanical counterparts. The actuators can buckle when the network assumes certain configurations. In order to carry heavy loads, the LAR must stay in configurations that provide the network with stiffness to gravitational forces. Therefore, it is important to control a LAR in a way that maximizes the network's stiffness to expected forces.

Previous work on the control of LARs has not focused on attaining any optimality conditions. In [10], distributed control is enabled by selecting a network topology in such a way to reduce the kinematic complexity. These simple topologies consist of a repeated graphical motif of a tetrahedron or an octahedron and can be described as minimally rigid – a property that simplifies the control and will be defined below. However, the control techniques used by [10] do not apply to arbitrary graph topologies. Other work has proposed simple control algorithms employing repetitive punctuated rolling gaits for locomotion of space exploration linear actuator robots [11]. More recently, [12] used differential kinematics to derive general solutions to kinematics and controllability for arbitrary network topologies.

In this work, I investigate how to control a LAR that maximizes the network's stiffness to gravitational forces and minimizes control input. This could have implications on the load carrying ability of the network, disturbance rejection during locomotion, and ability to assume certain target configurations. In Section II I will describe the model used to represent the LAR dynamics and the kinematic constraints. In Section III I will describe the optimal control approach and simulations showing their results. A discussion of the results is given in Section IV. Section V contains concluding remarks.

II. Model Formalization

A. Graph Theory

The linear actuator robot is mathematically represented as a framework consisting of a graph and vertex positions. The graph is denoted as $G = \{V, E\}$ where $V = \{1, ..., n\}$ is a set of vertices (or nodes) and $E \subset VxV$ is a set of edges. The graph is connected, undirected, unweighted and has a constant topology. In this paper, the vertices are embedded in three dimensional space, $\mathbb{R}^3$, and have position vectors $p_i = [p_{ix}, p_{iy}, p_{iz}]^T$. We will use the terminology of vertices and nodes interchangeably throughout this paper. We define a position vector of the entire framework as $p = [p_{1x}^T, ..., p_{nx}^T, p_{1y}^T, ..., p_{ny}^T, p_{1z}^T, ..., p_{nz}^T]^T$. The length of an edge of the graph is the euclidean distance between the nodes: $L_{ij} = \|v_j - v_i\|$.

B. Graph Rigidity

A rigid framework is defined as a framework where the only possible motions of the vertices that can occur without changes to the edge lengths are rigid body transformations of the entire framework. A more detailed mathematical definition of rigidity is given by Asimow and Roth, and will not be repeated here [13]. A minimally rigid graph is a rigid graph where the removal of any link causes the graph to lose rigidity. These minimally rigid graphs provide a lower bound on the number of links necessary to constrain a certain number of vertices. When a linear actuator robot is minimally rigid, the lengths of the actuators can be changed independently of one another. Increasing the number of actuators necessarily over constrains some of the vertices of the network. Therefore, the lengths of some linear actuators cannot be changed without other actuators changing as well. An infinitesimally rigid framework is rigid even to theoretically infinitesimal perturbations of the vertices. For examples of non-rigid frameworks, minimally rigid frameworks, rigid frameworks, and over-constrained frameworks, see Fig 3.

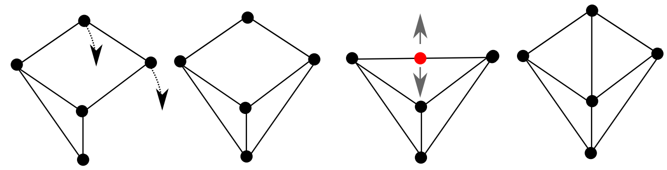

Fig. 3: (i) A non-rigid framework. Arrows show the direction nodes can be moved with no change to lengths. (ii) A minimally infinitesimally rigid network. (iii) The network has the same topology as (ii), and is rigid but not infinitesimally rigid, and hence not controllable. No controlled motion is possible in the direction of the arrows. (iv) An additional edge is added to (ii), meaning the structure is no longer minimally rigid. Motions of the actuators must be coordinated, and can not always be made independently.

C. Differential Kinematics

We can find the differential kinematics by taking the time derivative of the lengths of the edges:

$$\frac{dL_{i,j}^2}{dt} = 2L_{i,j}\dot{L}_{i,j} = 2(p_i - p_j)^T\dot{p}_i + 2(p_j - p_i)^T\dot{p}_j. \quad (1)$$

Rewriting in matrix form

$$\begin{bmatrix} \dot{L}_{i,j} \\ \dot{L}_{i,j} \\ \dots \\ \dot{L}_{i,j} \end{bmatrix} = R(x) \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_n \end{bmatrix}. \quad (2)$$

In this equation, $R(x)$ is a scaled version of the well known rigidity matrix [13,14]. If the matrix $R(x)$ is of maximum rank, we say the framework is infinitesimally rigid. While the properties rigidity and minimal rigidity are determined simply by the topology of the graph, infinitesimal rigidity is also dependent on the configuration x. Infinitesimally rigid frameworks are a subset of rigid frameworks, but the converse is not true.

D. Kinematic Constraints

Four types of physical constraints must be satisfied to guarantee feasibility of a given framework (G, p). The first of these constraints is that the lengths of all of the edges must fall within a fixed minimum and maximum length range that the linear actuators are capable of attaining. The actuator length, L_{ij}, is the length of the edge, e_{ij}, connecting vertices i, j and is computed as the euclidean distance between the two vertices. We can succinctly write this constraint using the graph laplacian, $\mathcal{L}$, and the Kronecker product:

$$L_{min}^2 \leq p^T[L \otimes I_d]p \leq L_{max}^2 \quad (3)$$

The second type of physical constraint is that the actuators cannot physically intersect each other. To determine if two actuators within a framework cross, the minimum distance between them must be greater than the diameter of the actuators, d_{min}. If the actuators do not have a cylindrical geometry, then d_{min} can be taken as the diameter of the smallest circle that can be inscribed by the actuators cross-sectional geometry. The minimum distance between actuators connecting vertices i, j, and k, l is:

$$d_{ij}^{kl} = min\|(p_i + \alpha(p_j - p_i) - (p_k + \gamma(p_l - p_k)\| \tag{4}$$

where $\alpha, \gamma \in (0, 1)$. The actuator collision constraint can now be written as:

$$d_{ij}^{kl} \geq d_{min} \forall \{i, j\}, \{k, l\} \in E \tag{5}$$

The third type of physical constraint is collision with the environment. The linear actuator robot is obviously unable to pass through obstacles it may encounter in the environment. We can describe the motion of the LAR with respect to the environments with the equation:

$$\dot{F} = C\dot{x} \tag{6}$$

where the C matrix relates the motion of the nodes ($\dot{x}$) with the changing environment ($\dot{F}$). The exact form of C can be determined based on how contact between the structure and the environment is modeled. For our purposes, we assume that nodes in contact with the environment do not move.Adding these constraints to our dynamics equations yields:

$$\begin{bmatrix} \dot{L} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} R \\ C \end{bmatrix} [\dot{x}] = H\dot{x}. \tag{7}$$

If the system is infinitesimally minimally rigid, and a minimal set of constraints is applied that is linearly independent of the link constraints, the combined matrix $[R^T C^T]^T$ is full rank and square, and hence invertible,

$$\dot{x} = H(x)^{-1} \begin{bmatrix} \dot{L} \\ \dot{F} \end{bmatrix}. \tag{8}$$

If the system is not minimally rigid, then the network is over-constrained and additional constraints on the lengths of the actuators are needed to ensure feasibility of the framework. This is the fourth type of constraint. In this case, the differential kinematics can be written as:

$$\dot{x} = H(x)_m^{-1} \begin{bmatrix} \dot{L_m} \\ \dot{F_m} \end{bmatrix} \tag{9}$$

$$s.t. \quad H_s(x) \begin{bmatrix} \dot{L} \\ \dot{F} \end{bmatrix} = 0. \tag{10}$$

Where L_m and F_m is the minimal set of input lengths needed for minimal rigidity which is related to the network state x by the matrix $H_m^{-1}(x)$. $H_s(x)$ describes the relationships between the minimal set of input lengths and the over-constraining lengths.

III. Optimal Controller

A. Problem Statement

In this section we examine how to control a LAR that satisfies optimality conditions for some given penalty function. Specifically of interest here is how to control a LAR that employs a punctuated rolling type gait in a manner that maximizes the networks stiffness to gravitational forces and minimizes control input. I identify two problems that are relevant. The first is moving the network from one configuration to another under an optimal control policy. The second is to select an optimal configuration the network should assume for tipping. Combing these two problem together, we will find a set of control inputs that drives the LAR from any feasible initial condition to the optimal tipping configuration in a manner that minimizes a penalty function based on the network's stiffness and control input.

B. Optimal Control Policy

To find an optimal control policy, we use a variational approach to the optimal control problem [15]. We seek an admissible control input u^* for a system subjected to the dynamics

$$\dot{x} = H(x)^{-1}u. \tag{11}$$

which is described in more detail in Section II. This optimal admissible control input should drive the system to follow an admissible trajectory x^* that minimizes our performance measure:

$$\begin{aligned} J \quad = \quad & \int_{t_0}^{t_f} [g^T S(x)^{-T} S(x)^{-1} g \\ & + \ \lambda u^T H(x)^{-T} H(x)^{-1} u] dt \end{aligned} \tag{12}$$

In the equation above, $S(x)$ is the well known stiffness matrix from the direct stiffness method [16], g is the expected force acting upon the LAR like gravity, and λ is a scalar weighting. We then formulate the following Hamiltonian equation:

$$\begin{aligned} \mathcal{H} \quad = \quad & g^T S(x)^{-T} S(x)^{-1} g \\ & + \lambda u^T H(x)^{-T} H(x)^{-1} u \\ & + p^T H^{-1} u \end{aligned} \tag{13}$$

The necessary optimality conditions are

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial p} \tag{14}$$

$$\dot{p} = \frac{\partial \mathcal{H}}{\partial x} \tag{15}$$

$$0 = \frac{\partial \mathcal{H}}{\partial u} \tag{16}$$

A two point boundary value problem can now be used to solve the system of differential equations (14) - (16). It should be noted that this analysis is able to incorporate the equality constraints but does not capture the inequality constraints discussed in Section II.

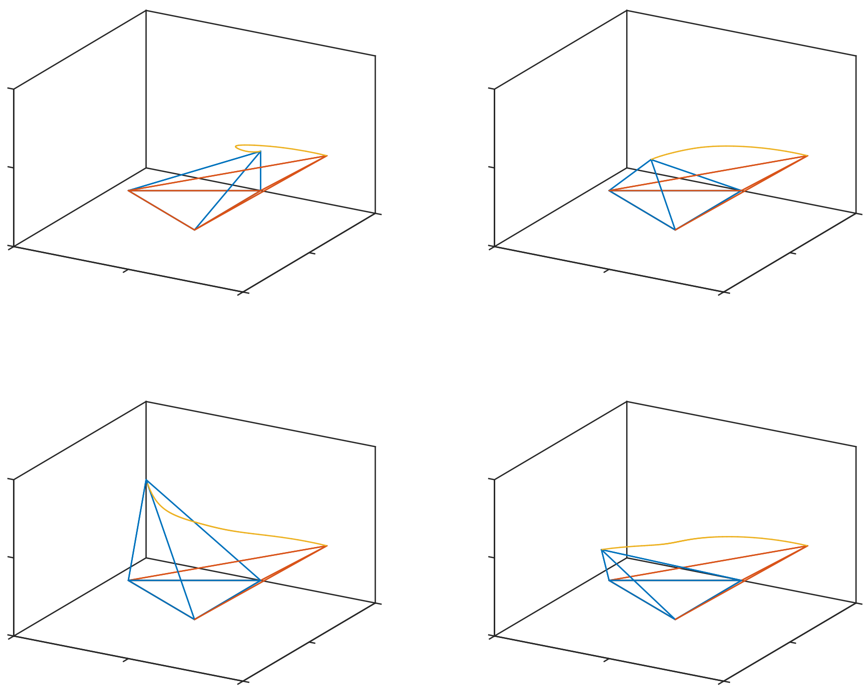

Fig. 4: Optimal trajectories for the tetrahedral LAR. The LAR is shown in its initial configuration (blue) and its optimal tipping configuration (red). The path of the unconstrained node is shown in orange.

C. Optimal Tipping Configuration

The second problem of finding the optimal tipping configuration will now be discussed. For the LAR to initiate a tip, the center of mass of the LAR must exit the support polygon, which is formed by the convex hull of the nodes of the framework on the ground plane. The center of mass of the LAR should exit the face of the support polygon that is perpendicular to the desired direction of motion. If we assume a lumped-mass model whereby the mass of the LAR is concentrated at the nodes, we can express the center of mass constraint as:

$$x_{COM} = Mx \tag{17}$$

where M is a $\mathbb{R}^{3x3n}$ mass matrix. The objective we will minimize is the total deformation due to a unit force applied to the network as it tips. The direction of this force is dependent on the network orientation on impact with the ground. We can write this optimization problem as:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|S(x)f\| \\ \text{subject to} \quad & \\ & x_{COM} = Mx \\ & L_{min}^2 \leq p^T[L \otimes I_d]p \leq L_{max}^2 \\ & d_{ij}^{kl} \geq d_{min} \; \forall \; \{i,j\},\{k,l\} \in E \\ & \dot{F} = C\dot{x} \\ & H_s(x)\begin{bmatrix}\dot{L} \\ \dot{F}\end{bmatrix} = 0 \end{aligned} \tag{18}$$

D. Simulation

The optimization problems discussed in Section III-B and Section III-C were simulated for a single tetrahedral LAR. Optimal control policies were determined for the tetrahedral structure that bring the LAR into the optimal tipping configuration. A few examples of the optimal trajectories found by this method are shown in Fig. 4.

IV. Discussion

While the methods proposed in Section III work efficiently for the single tetrahedral structure shown in Fig. 4 more work needs to be done to fully characterize the performance of the method on frameworks of varying topology. The single tetrahedral LAR framework is the most basic three dimensional case. As more actuators and vertices are added to the LAR the framework may lose minimal rigidity and the configuration space will grow significantly. There will be an increasing need to fully represent all constraints and more local minima with jeopardize our ability to find a global minimum. This will drive the need to apply a direct method approach to finding the optimal control policy that can include the inequality constraints imposed by the robot kinematics.

This paper focuses on network stiffness for the optimality criterion. However, it may be interesting to consider other metrics for optimal control. For example, one may want a solution that encourages sparsity, minimum completion time, or smoothest trajectory.

V. Conclusion

We have presented methods to optimally control LAR configuration and determine optimal tipping configurations that maximize the stiffness of the framework to external forces. Although there is still much to improve on these techniques, the methods proposed in this paper are the first time LAR control policies have been placed within an optimal control framework. This preliminary work encourages future work on optimal control of LARs.

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